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Spin-dependent nuclear structure functions: general approach with application to the Deuteron *

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Abstract

We study deep-inelastic scattering from polarized nuclei within a covariant framework. A clear connection is established between relativistic and non-relativistic limits, which enables a rigorous derivation of convolution formulae for the spin-dependent nuclear structure functions g_1^A and g_2^A in terms of off-mass-shell extrapolations of polarized nucleon structure functions, g_1^N and g_2^N. Approximate expressions for $g_{1,2}^A$ are obtained by expanding the off-shell $g_{1,2}^N$ about their on-shell limits. As an application of the formalism we consider nuclear effects in the deuteron, knowledge of which is necessary to obtain accurate information on the spin-dependent structure functions of the neutron.

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I. INTRODUCTION

Polarized deep-inelastic scattering (DIS) experiments have in recent years yielded a number of important and sometimes unexpected results. The measurements by the European Muon Collaboration (EMC) of the g_1^p structure function of the proton [1] over a large range of values of the Bjorken scaling variable x, when combined with flavor non-singlet matrix elements from weak decays, provided information on the singlet axial charge of the proton. The small size of this resulted in the so-called proton "spin crisis", which prompted a serious reanalysis of the very ideas behind the quark model and the simple parton picture of DIS. More recent experiments on the proton by the Spin Muon Collaboration (SMC) [2] and the SLAC E143 Collaboration [3] have allowed more refined analyses of the x and Q^2 dependence of the g_1^p structure function [4–6]. (For a recent review see Ref. [7].)

As an extra source of information, it is important also to measure the neutron polarized structure function, g_1^n. Besides revealing the spin structure of the neutron itself, measurement of g_1^n is essential for testing the fundamental Bjorken sum rule. The absence of free neutron targets means, however, that light nuclei have to be used instead for this purpose. The SLAC E142 Collaboration [8] has in fact measured the structure function of ^{3}He, which, because of the preferential antiparallel polarization of protons in the ^{3}He nucleus, is believed to be approximately equal to the polarized structure function of the neutron. In addition, the SMC [9] has recently measured the g_1 structure function of the deuteron — combined with either the proton or neutron (^{3}He) data, this can be used as a valuable consistency check on the other measurements.

To obtain accurate information on nucleon structure functions from nuclear DIS data, it is of course essential to reliably subtract any nuclear effects in the extraction procedure. Away from the shadowing region at small Bjorken x ($x \lesssim 0.1$), the standard method for investigating nuclear effects is the so-called convolution model, in which the nuclear structure function is expressed as a (one-dimensional) convolution of the spin-dependent nucleon structure function and the nucleon momentum distribution in the nucleus. The convolution model follows from the impulse approximation, Fig.1, if one assumes that factorization between photon–nucleon and nucleon–nucleus scattering amplitudes translates into factorization between structure functions, although in a covariant framework this assumption is generally not justified.

For ^{3}He targets, nuclear effects have been investigated in Refs. [10,11], and for the deuteron in Refs. [10,12–17]. In Refs. [15,17] relativistic effects in the deuteron were also included, although still within the confines of the convolution model. It was shown in Ref. [16], however, that relativistic corrections necessarily lead to a breakdown of convolution for g_1^A when the full off-mass-shell structure of bound nucleons is incorporated. Although the convolution-breaking effects are not large (typically $\sim 0.5\%$ in the deuteron [16]), in any self-consistent calculation they must be included.

Aside from the relativistic complications, the situation is not completely clear even in the non-relativistic approaches. There exists in the literature [10–14] a variety of results for convolution formulae for g_1^A, the derivation of which is often based on early convolution models for unpolarized scattering [18], in which the issue of off-shell effects was not seriously addressed. The need exists, therefore, to derive convolution formulae for spin-dependent structure functions systematically in the non-relativistic limit.

In this paper we present an analysis of the polarized structure functions of nuclei, starting from a covariant framework, and working consistently to order $\mathbf{p}^2/M^2$ in the bound nucleon momentum. We demonstrate that in this limit one does indeed recover (two-dimensional) convolution formulae, although with different "flux factors" (polarized nucleon momentum distributions) compared to those found in the literature. Our formalism enables us to consider both the g_1 and g_2 structure functions (or equivalently the transverse structure function $g_T \equiv g_1 + g_2$) on a similar footing. For the latter, we find that g_2^A receives contributions from the g_1^N as well as from the g_2^N structure functions of the nucleon. All of the formal results are valid in the Bjorken limit for spin 1/2 and spin 1 nuclei.

Taking advantage of the weak binding of nucleons in nuclei, we also derive expansion formulae for $g_{1,T}^A$ in terms of derivatives of $g_{1,T}^N$. We concentrate on the specific case of the deuteron, where we present a detailed comparison of the expansion formula results with those of the full non-relativistic convolution, as well as with previous relativistic calculations. In addition, we estimate for the first time the nuclear effects that one needs to account for when separating the nucleon g_T (or g_2) structure function from deuteron data. This will be important in view of upcoming experiments which will for the first time measure g_2 for deuterium targets.

The contents of this paper are laid out as follows: in Section II we present the general covariant framework in which $g_{1,T}^A$ of a nucleus A are expressed in terms of the off-shell nucleon propagator and the virtual nucleon hadronic tensor; in Section III we describe how the relativistic expressions can be reduced by making a non-relativistic expansion of the nucleon propagator in medium; Section IV deals with the details of approximate expansion formulae which are obtained by Taylor-expanding the off-shell nucleon structure function about its on-shell limit; application of the formalism to the case of deuterium is presented in Section V; finally concluding remarks are made in Section VI.

II. RELATIVISTIC FRAMEWORK

A. Definitions

Inclusive deep-inelastic scattering of leptons from hadrons is described by the hadronic tensor

$$W_{\mu\nu}(P,q,S) = \frac{1}{2\pi} \int d^4\xi\, e^{iq\cdot\xi} \left\langle P,S \left| [J_\mu(\xi), J_\nu(0)] \right| P,S \right\rangle , \tag{1}$$

where P and q are the four-momenta of the target and photon, respectively, and the vector S is orthogonal to the target momentum, $P\cdot S = 0$, and normalized such that $S^2 = -1$. For spin-1/2 targets, S is simply the target polarization vector, while for the spin-1 case S is defined in terms of polarization vectors ε_α^m such that $S^\alpha(m) = -i\epsilon^\alpha\left(\varepsilon^{m*}, \varepsilon^m, P\right)/M_T$, where $m = 0, \pm 1$ is the spin projection along the axis of quantization, M_T denotes the target mass, and we define $\epsilon^\alpha(a,b,c) \equiv \epsilon^{\alpha\beta\mu\nu} a_\beta b_\mu c_\nu$. The hadronic tensor can be decomposed into symmetric (s) and antisymmetric (a) parts,

$$W_{\mu\nu}(P,q,S) = W_{\mu\nu}^{(s)}(P,q,S) \;+\; i\, W_{\mu\nu}^{(a)}(P,q,S). \tag{2}$$

With unpolarized (charged) lepton beams one is sensitive only to the symmetric part, which, at leading twist, depends on the spin-independent F_1 and F_2 structure functions, and, for spin-1 targets, also on the structure functions $b_{1,2}$ when the target alone is polarized [19]. If the lepton and hadron are both polarized, only the antisymmetric component of $W_{\mu\nu}$ is relevant. For either spin-1/2 or spin-1 targets this is expressed in terms of the two independent, dimensionless structure functions $g_{1,2}$:

$$W_{\mu\nu}^{(a)}(P,q,S) = \frac{1}{2P\cdot q}\epsilon_{\mu\nu\alpha\beta}\, q^\alpha\, G^\beta(P,q,S), \tag{3}$$

$$G^\beta(P,q,S) = 2M_T\left[S^\beta\,(g_1+g_2) - P^\beta\frac{S\cdot q}{P\cdot q}g_2\right]. \tag{4}$$

In the Bjorken limit ($Q^2, P\cdot q \to \infty$), in which we work throughout, both structure functions g_1 and g_2 exhibit scaling, i.e. up to logarithmic QCD corrections they depend only on the ratio $Q^2/2P\cdot q$.

B. Nucleon Tensor and Structure Functions

In our covariant analysis it will be useful to work with the off-shell nucleon tensor $\widehat{\mathcal{W}}_{\mu\nu}$, which is defined through the imaginary part of the forward photon scattering amplitude from an off-shell nucleon. In terms of $\widehat{\mathcal{W}}_{\mu\nu}$, the hadronic tensor of an on-shell nucleon ($p^2 = M^2$ where p is the nucleon momentum) is:

$$W_{\mu\nu}^N(p,q,s) = \mathrm{Tr}\left[(\not p + M)\frac{(1+\gamma_5\not s)}{2}\;\widehat{\mathcal{W}}_{\mu\nu}(p,q)\right], \tag{5}$$

where s is the nucleon spin vector, and M is the nucleon mass. The antisymmetric part of $\widehat{\mathcal{W}}_{\mu\nu}$ is given by an expression similar to that in Eq.(3), namely:

$$\widehat{\mathcal{W}}_{\mu\nu}(p,q) = \frac{1}{2p\cdot q}\epsilon_{\mu\nu\alpha\beta}\, q^\alpha\, \widehat{\mathcal{G}}^\beta(p,q). \tag{6}$$

In analyzing the tensor structure of $\widehat{\mathcal{W}}_{\mu\nu}$ it is convenient to expand $\widehat{\mathcal{G}}^\beta$ in terms of a complete set of Dirac matrices, $\left\{I, \gamma^\alpha, \sigma^{\alpha\beta}, \gamma^\alpha\gamma_5, \gamma_5\right\}$. The various coefficients in this expansion must be constructed from the vectors p and q, and from the tensors $g_{\alpha\beta}$ and $\epsilon_{\mu\nu\alpha\beta}$. Terms proportional to q^β do not contribute to $\widehat{\mathcal{W}}_{\mu\nu}$, and are therefore not considered. The requirements of parity, time-reversal invariance and hermiticity restrict further the number of possible terms. In particular, it follows from parity invariance that $\widehat{\mathcal{G}}^\beta$ transforms as an axial vector and the coefficient of the scalar (I) term must be zero. The requirements of time reversal invariance and hermiticity rule out the vector (γ^α) and the pseudoscalar (γ_5) terms as well. Finally we find that $\widehat{\mathcal{G}}^\beta$ can be written in terms of six independent structures:

$$\begin{aligned}\widehat{\mathcal{G}}^\beta(p,q) = {} & \left(\frac{G^{(p)}}{M^2}\not p + \frac{G^{(q)}}{p\cdot q}\not q\right) p^\beta\gamma_5 + G^{(\gamma)}\gamma^\beta\gamma_5 \\ & + \left(\frac{G^{(\sigma p)}}{M}\,p_\alpha + \frac{G^{(\sigma q)}}{p\cdot q}\,q_\alpha\right) i\gamma_5\sigma^{\beta\alpha} + \frac{G^{(\sigma pq)}}{M\;p\cdot q}\,p^\beta i\gamma_5\sigma^{\alpha\lambda}\,p_\alpha q_\lambda,\end{aligned} \tag{7}$$

where the coefficient functions $G^{(i)}$ are constructed to be scalar, dimensionless and real functions of q^2, $p \cdot q$ and p^2.

Substituting $\widehat{\mathcal{G}}^\beta$ into Eq.(5) we can express the on-shell nucleon [1] structure functions $g_{1,2}^N$ in terms of the coefficients $G^{(i)}$ (see also Ref. [16]):

$$g_1^N = G^{(q)} + G^{(\gamma)} + G^{(\sigma p)} - G^{(\sigma p q)}, \tag{8a}$$

$$g_2^N = -G^{(q)} + G^{(\sigma q)} + G^{(\sigma p q)}, \tag{8b}$$

with the functions $G^{(i)}$ evaluated here at their on-shell values. Note that the term proportional to $\not{p}\gamma_5$ vanishes when tracing $\widehat{\mathcal{G}}^\beta$ with the projection operator in Eq.(5), so that only five terms out of a possible six in Eq.(7) contribute to the physical nucleon structure functions. The structure $\not{p}\gamma_5$ could, in general, give non-vanishing contribution to structure functions of nuclei. However, as we shall see in Section III, only the above five functions will be relevant in the non-relativistic limit.

C. Nuclear Structure Functions

Discussions of nuclear effects in deep-inelastic scattering are usually framed within the context of the impulse approximation for the nucleons, see Fig.1. Other possible nuclear effects which go beyond the impulse approximation are final state interactions between the recoiling nucleus and the debris of the struck nucleon [20], corrections due to mesonic exchange currents [21–23], and nuclear shadowing [2]. One may argue that complications due to meson exchange currents are less important here than in unpolarized scattering since their main contribution comes from pion exchange. Because it has spin zero, direct scattering from a pion constituent of a nucleus gives no contribution to spin-dependent structure functions. Also coherent multiple scattering effects, which are known to lead to nuclear shadowing, should not be important for large values of the nucleon Bjorken scaling variable $x = Q^2/2Mq_0$. This is evident if one recognizes that the characteristic time scale $1/Mx$ of the DIS process is smaller than the typical average distance between bound nucleons in the nucleus for $x > 0.1$. Based on these observations we consider the diagram in Fig.1 as a basic approximation. In this case the nuclear hadronic tensor can be written:

$$W_{\mu\nu}^A(P, q, S) = \int [dp] \ \mathrm{Tr} \left[\mathcal{A}(p; P, S) \ \widehat{\mathcal{W}}_{\mu\nu}(p, q) \right], \tag{9}$$

with $[dp] \equiv d^4p/(2\pi)^4 i$. The function $\mathcal{A}(p; P, S)$ is the nucleon propagator inside the nucleus with momentum P and polarization S,

$$\mathcal{A}(p; P, S) = -i \int d^4\xi \, e^{ip\cdot\xi} \left\langle P, S \left| T \left(N(\xi)\overline{N}(0) \right) \right| P, S \right\rangle . \tag{10}$$

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[1]Throughout we define $g_{1,2}^N$ to be the average of the proton and neutron structure functions: $g_{1,2}^N = \left(g_{1,2}^p \ + \ g_{1,2}^n \right) \Big/ \, 2$.

[2]Some potential problems associated with the use of the impulse approximation for the g_2 structure function have also been discussed in Ref. [24] in the context of relativistic light-front dynamics.

Here $N(\xi)$ is the nucleon field operator, and $\widehat{\mathcal{W}}_{\mu\nu}(p,q)$ is the hadronic tensor of the off-mass-shell nucleon, given by Eqs.(6) and (7).

The expression for the nuclear tensor in Eq.(9) is covariant and can be evaluated in any frame. It will be convenient, however, to work in the target rest frame, in which the target momentum is $P = (M_A; \mathbf{0})$, and the momentum transfer to the nucleus, $q = (q_0; \mathbf{0}_\perp, -|\mathbf{q}|)$, defines the z-axis. For the g_1 structure function it is natural to choose the spin quantization axis such that the target is polarized in a direction longitudinal to the momentum transfer, $S = S^\parallel = (0; \mathbf{0}_\perp, 1)$. Taking the W^A_{12} component in Eq.(9), which is proportional to g^A_1, we find:

$$x_A g_1^A(x_A) = \frac{1}{2M_A} \int [dp]\ x'\ \mathrm{Tr}\left[\mathcal{A}^\parallel(p)\ \left(\widehat{\mathcal{G}}_0 + \widehat{\mathcal{G}}_z\right)\right], \tag{11a}$$

where $x_A = Q^2/2P\cdot q$ and $x' = Q^2/2p\cdot q$ are the Bjorken variables for the nucleus and bound (off-mass-shell) nucleon, respectively. The nucleon propagator in the nucleus at rest with polarization $S^\parallel$ is denoted by $\mathcal{A}^\parallel(p) = \mathcal{A}(p; S^\parallel)$.

The transverse spin-dependent structure function of the nucleus, $g^A_T \equiv g^A_1 + g^A_2$, is obtained by choosing the target polarization in a direction perpendicular to the momentum transfer, $S = S^\perp = (0; \mathbf{S}^\perp, 0)$, and taking the W^A_{13} and W^A_{23} components in Eq.(9):

$$x_A g_T^A(x_A) = \frac{1}{2M_A} \int [dp]\ x'\ \mathrm{Tr}\left[\mathcal{A}^\perp(p)\, \widehat{\mathcal{G}}_\perp\right], \tag{11b}$$

where $\mathcal{A}^\perp(p) = \mathcal{A}(p; S^\perp)$, and $\widehat{\mathcal{G}}_\perp$ represents the transverse spatial components of $\widehat{\mathcal{G}}^\beta$, corresponding to the transverse quantization axis. From Eqs.(11a) and (11b) one can reconstruct the g^A_2 structure function by taking the difference between g^A_T and g^A_1.

We should stress that the treatment culminating in the results of Eqs.(11) has been fully relativistic, and exact within the impulse approximation. In the literature one usually encounters formulations in terms of simple convolution formulae [10–15,17], in which the nuclear structure functions are expressed as one-dimensional convolutions of the nucleon momentum distribution in the nucleus, and the (on-shell) structure functions of the nucleon. To obtain the simple convolution result one requires two conditions to be satisfied: firstly, that the traces $\mathrm{Tr}\,[\mathcal{A}\,\widehat{\mathcal{G}}]$ in Eqs.(11) factorize into completely separate nuclear and nucleon parts, and secondly that the nucleon component be independent of p^2. For the twist-2 structure function g^A_1, it was shown in Ref. [16] that in a relativistic treatment the off-shell degrees of freedom associated with bound nucleons in fact violate both conditions, leading to a breakdown of the simple convolution picture. A similar breakdown must also occur for the g^A_2 structure function, since its twist-2 contribution contains a component proportional to g^N_1 (see Section V A below).

Clearly, in a relativistic theory one needs to go beyond the simple convolution formulation. On the other hand, the advantage of the convolution model is its ease of application. In the study of DIS from nuclei, especially light nuclei where typical binding energies are small, it may in fact be quite sufficient to treat the nucleus as a non-relativistic system. It was shown in Ref. [25] that for unpolarized nuclear structure functions, in which both of the above criteria are also violated relativistically [26,27], one can in fact recover factorized expressions in the non-relativistic limit. It is possible to define "off-shell nucleon structure functions", with the correct on-shell limits, which lead to (two-dimensional) convolution

formulae. In the next section we perform a similar non-relativistic reduction of the relativistic expressions in Eqs.(11) to establish whether a similar factorization is attainable in spin-dependent processes.

III. NUCLEAR STRUCTURE FUNCTIONS IN THE NON-RELATIVISTIC LIMIT

Our basic assumption in the remainder of the paper is that the nucleus is a non-relativistic system, made up of weakly bound nucleons interacting via the exchange of mesonic fields. This necessarily involves neglecting antinucleon degrees of freedom, and corresponds to bound nucleons in the nucleus being slow, $|\mathbf{p}| \ll M$. With these assumptions we will derive from Eqs.(11) simplified (convolution) expressions for the spin-dependent $g_{1,T}^A$ structure functions of weakly bound nuclei.

A. Non-Relativistic Reduction

Following the procedure outlined for example in Ref. [28], we can derive the relation between the relativistic nucleon field operator N and the non-relativistic operator ψ. A detailed discussion of the non-relativistic reduction of N is given in Appendix A. The essential result is that, up to order $\mathbf{p}^3/M^3$ corrections, the operators N and ψ are connected via:

$$N(\mathbf{r},t) = e^{-iMt} \begin{pmatrix} Z\,\psi(\mathbf{r},t) \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{2M}\,\psi(\mathbf{r},t) \end{pmatrix}, \tag{12}$$

where $\mathbf{p} \equiv -i\boldsymbol{\nabla}$, and the renormalization operator $Z = 1 - \mathbf{p}^2/8M^2$ guarantees baryon number conservation. This result is valid for a wide range of meson–nucleon interactions, in particular for interactions with scalar and vector mesons as well as for pseudovector coupling to pions. Furthermore, it is explicitly interaction–independent. For the pseudoscalar πN couplings discussed in Refs. [22,29], however, one finds explicit interaction dependence in Eq.(12), although the pseudoscalar interaction is generally considered less reliable than the pseudovector model, which we restrict ourselves to in this paper (see Appendix A).

Consider now the consequences of applying Eq.(12) to the traces in Eqs.(11). We start by writing the nucleon propagator (10) as:

$$\frac{1}{2M_A}\mathcal{A}_{\alpha\beta}(p) = -i\int dt\, e^{ip_0 t}\left\langle T\left(N_\alpha(\mathbf{p},t)\overline{N}_\beta(\mathbf{p},0)\right)\right\rangle, \tag{13}$$

where $N_\alpha(\mathbf{p},t) = \int d^3\mathbf{r}\, e^{-i\mathbf{p}\cdot\mathbf{r}} N_\alpha(\mathbf{r},t)$ is the nucleon operator in a mixed $(\mathbf{p},t)$-representation, and the brackets denote an average over the nuclear state, $\langle\cdots\rangle \equiv \langle A|\cdots|A\rangle\,/\,\langle A|A\rangle$. Writing the four-momentum of the bound nucleon as $p = (M+\varepsilon;\mathbf{p})$, we can introduce the non-relativistic nucleon propagator $\mathcal{A}^{\rm NR}$, which is usually defined as:

$$\mathcal{A}^{\rm NR}_{\sigma\sigma'}(\varepsilon,\mathbf{p}) = -i\int dt\, e^{i\varepsilon t}\left\langle T\left(\psi_\sigma(\mathbf{p},t)\psi^\dagger_{\sigma'}(\mathbf{p},0)\right)\right\rangle, \tag{14}$$

where σ, σ' are the non-relativistic, two-dimensional nucleon spinor indices. The relativistic and non-relativistic nucleon propagators can then be related by substituting Eq.(12) into Eq.(13):

$$\frac{1}{2M_A}\mathcal{A}_{\alpha\beta}(p) = U_{\alpha\sigma}\ \mathcal{A}^{\rm NR}_{\sigma\sigma'}(\varepsilon,\mathbf{p})\ U^\dagger_{\sigma'\alpha'}\gamma^0_{\alpha'\beta}. \tag{15}$$

Here the operator U translates two-component spinors into four-component spinors:

$$U_{\alpha\sigma}\psi_\sigma = \begin{pmatrix} Z\,\psi \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{2M}\,\psi \end{pmatrix}_\alpha . \tag{16}$$

Equation (15) can be used to reduce the traces of the relativistic propagator $\mathcal{A}$ with the various Dirac structures in $\widehat{\mathcal{G}}^\beta$ to expressions involving the non-relativistic propagator $\mathcal{A}_{\rm NR}$:

$$\frac{1}{2M_A}{\rm Tr}\left[\gamma_0\gamma_5\mathcal{A}(p)\right] = {\rm tr}\left[\widehat{\mathcal{S}}_0\ \mathcal{A}_{\rm NR}(\varepsilon,\mathbf{p})\right], \tag{17a}$$

$$\frac{1}{2M_A}{\rm Tr}\left[\gamma_j\gamma_5\mathcal{A}(p)\right] = {\rm tr}\left[\widehat{\mathcal{S}}_j\ \mathcal{A}_{\rm NR}(\varepsilon,\mathbf{p})\right], \tag{17b}$$

$$\frac{1}{2M_A}{\rm Tr}\left[i\gamma_5\sigma_{0j}\mathcal{A}(p)\right] = {\rm tr}\left[\left(-\sigma_j + \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{2M^2}p_j\right)\mathcal{A}_{\rm NR}(\varepsilon,\mathbf{p})\right], \tag{17c}$$

$$\frac{1}{2M_A}{\rm Tr}\left[i\gamma_5\sigma_{ij}\mathcal{A}(p)\right] = {\rm tr}\left[\left(\frac{\sigma_i p_j - \sigma_j p_i}{M}\right)\mathcal{A}_{\rm NR}(\varepsilon,\mathbf{p})\right], \tag{17d}$$

where

$$\widehat{\mathcal{S}}_0 = \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{M}, \tag{18a}$$

$$\widehat{\mathcal{S}}_j = \left(1 - \frac{\mathbf{p}^2}{2M^2}\right)\sigma_j\ +\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{2M^2}\ p_j, \tag{18b}$$

and i, j denote spatial indices. The trace "tr" is taken with respect to the spin variable in two-component space, ${\rm tr}\,\mathcal{O} \equiv \mathcal{O}_{\sigma\sigma}$. All corrections to Eqs.(17) are of order $|\mathbf{p}|^3/M^3$ or higher. One notices that the operators $\widehat{\mathcal{S}}_{0,j}$ have a structure similar to the time and space components of the spin four-vector $(0;\boldsymbol{\sigma})$ boosted to a frame in which the nucleon has momentum $\mathbf{p}$.

From Eqs.(17) and (18) one can determine the traces relevant for $g^A_{1,T}$ in Eqs.(11), namely:

$$\begin{aligned}
&\frac{1}{2M_A}{\rm Tr}\left[\mathcal{A}^{\parallel}(p)\ \left(\widehat{\mathcal{G}}_0 + \widehat{\mathcal{G}}_z\right)\right] \\
&= \left(G^{(q)} + G^{(\gamma)} + \left(G^{(\sigma p)} - G^{(\sigma pq)}\right)\left(1 + \frac{p^2 - M^2}{2M^2}\right)\right){\rm tr}\left[\mathcal{A}^{\parallel}_{\rm NR}(\varepsilon,\mathbf{p})\ \left(\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z\right)\right],
\end{aligned} \tag{19a}$$

$$\begin{aligned}
&\frac{1}{2M_A}{\rm Tr}\left[\mathcal{A}^{\perp}(p)\,\widehat{\mathcal{G}}_\perp\right] \\
&= \left(G^{(\gamma)} + G^{(\sigma p)}\left(1 + \frac{p^2 - M^2}{2M^2}\right) + G^{(\sigma q)}\left(1 - \frac{p^2 - M^2}{2M^2}\right)\right){\rm tr}\left[\mathcal{A}^{\perp}_{\rm NR}(\varepsilon,\mathbf{p})\,\widehat{\mathcal{S}}_\perp\right] \\
&+ \left(-G^{(q)} + G^{(\sigma q)}\left(1 - \frac{p^2 - M^2}{2M^2}\right) + G^{(\sigma pq)}\left(1 + \frac{p^2 - M^2}{2M^2}\right)\right){\rm tr}\left[\mathcal{A}^{\perp}_{\rm NR}(\varepsilon,\mathbf{p})\,\widehat{\mathcal{T}}_2\right],
\end{aligned} \tag{19b}$$

where $\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z$ and $\widehat{\mathcal{S}}_\perp$ are given by (18), and $p^2 \approx M^2 + 2M(\varepsilon - \mathbf{p}^2/2M)$ is the squared nucleon four-momentum (the ε^2 term is dropped since it introduces corrections of order $\mathbf{p}^4/M^4$). The operator $\widehat{\mathcal{T}}_2$ is given by:

$$\widehat{\mathcal{T}}_2 = -\frac{\mathbf{p}_\perp \cdot \mathbf{S}^\perp}{M}\left(\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{M} + \sigma_z\left(1 - \frac{p_z}{M}\right)\right), \tag{20}$$

where $\mathbf{p}_\perp$ is the transverse component of the nucleon three-momentum vector: $\mathbf{p} = (\mathbf{p}_\perp, p_z)$, and $\mathbf{S}^\perp$ defines the transverse spin quantization axis, relative to the photon direction (not to be confused with the spin operator $\widehat{\mathcal{S}}_\perp$). Note that in the non-relativistic limit the structure $G^{(p)}$ does not contribute to (19).

An important observation which can be made from Eq.(19) is that the nuclear structure functions are expressed in terms of only <u>two</u> combinations constructed from the $G^{(i)}$ in Eq.(7),

$$g_1^N(x,p^2) = G^{(q)} \ + \ G^{(\gamma)} \ + \ \left(G^{(\sigma p)} - G^{(\sigma p q)}\right)\left(1 + \frac{p^2 - M^2}{2M^2}\right), \tag{21a}$$

$$g_T^N(x,p^2) = G^{(\gamma)} \ + \ G^{(\sigma p)}\left(1 + \frac{p^2 - M^2}{2M^2}\right) \ + \ G^{(\sigma q)}\left(1 - \frac{p^2 - M^2}{2M^2}\right), \tag{21b}$$

so that also

$$\begin{aligned} g_2^N(x,p^2) &= -G^{(q)} \ + \ G^{(\sigma q)}\left(1 - \frac{p^2 - M^2}{2M^2}\right) \ + \ G^{(\sigma p q)}\left(1 + \frac{p^2 - M^2}{2M^2}\right) \\ &= g_T^N(x,p^2) \ - \ g_1^N(x,p^2). \end{aligned} \tag{21c}$$

These can be considered as *definitions* of the polarized nucleon structure functions in the off-mass-shell region (c.f. Eqs.(8) above) in the vicinity of $p^2 \approx M^2$. Note that in the $p^2 = M^2$ limit they reduce directly to the free nucleon structure functions defined in Eqs.(8).

B. Two-Dimensional Convolution

The definitions of the off-shell structure functions in Eqs.(21) can now be utilized in deriving convolution formulae for $g_{1,T}^A$. After substituting Eqs.(19) into (11), we make use of the analytical properties of the nucleon propagator for the integration over the momentum p. Namely, we close the contour of integration in the upper half of the complex ε (or p_0) plane and pick the poles of $\mathcal{A}_{\rm NR}(\varepsilon)$ which correspond to the time ordering $\theta(-t)\left\langle \psi^\dagger(0)\psi(t)\right\rangle$. Finally, one obtains simplified versions of Eqs.(11) which relate the polarized nuclear structure functions directly to those of nucleons:

$$xg_1^A(x) = \int\frac{d\varepsilon d^3\mathbf{p}}{(2\pi)^4}\ \mathrm{tr}\left[\mathcal{P}^{\parallel}(\varepsilon,\mathbf{p})\ \left(\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z\right)\right] x' g_1^N(x',p^2)\ , \tag{22a}$$

$$xg_T^A(x) = \int\frac{d\varepsilon d^3\mathbf{p}}{(2\pi)^4}\ \left(\mathrm{tr}\left[\mathcal{P}^{\perp}(\varepsilon,\mathbf{p})\,\widehat{\mathcal{S}}_\perp\right] x' g_T^N(x',p^2) \ + \ \mathrm{tr}\left[\mathcal{P}^{\perp}(\varepsilon,\mathbf{p})\,\widehat{\mathcal{T}}_2\right] x' g_2^N(x',p^2)\right), \tag{22b}$$

where here the nuclear structure functions are expressed as functions of the standard Bjorken variable $x = Q^2/2Mq_0 = x_A M_A/M$. For a general polarization state, the nuclear spectral function, $\mathcal{P}(\varepsilon,\mathbf{p})$, is defined by:

$$\mathcal{P}_{\sigma\sigma'}(\varepsilon, \mathbf{p}) = \sum_n \psi_{n,\sigma}(\mathbf{p}) \psi^*_{n,\sigma'}(\mathbf{p}) \;\; 2\pi\delta\left(\varepsilon - E_0(A) + E_n(A-1, -\mathbf{p})\right), \tag{23}$$

where the summation is performed over the complete set of states with $A-1$ nucleons. The functions $\psi_{n,\sigma}(\mathbf{p}) = \langle (A-1)_n, -\mathbf{p} | \psi_\sigma(0) | A \rangle$ give the probability amplitude to find in the nuclear ground state a nucleon with polarization σ and the remaining $A-1$ nucleons in a state with total momentum $-\mathbf{p}$ (n labels all other quantum numbers). The non-relativistic energies of the target ground state and the $A-1$ residual nucleons are denoted $E_0(A)$ and $E_n(A-1)$, respectively. Summing over polarizations σ and σ', the spectral function $\mathcal{P}(\varepsilon, \mathbf{p})$ is normalized to the number of nucleons A:

$$\int \frac{d\varepsilon d^3\mathbf{p}}{(2\pi)^4} \; \mathrm{tr}\left[\mathcal{P}(\varepsilon, \mathbf{p})\right] = A. \tag{24}$$

Equations (22) can be written in a more familiar form as two-dimensional convolutions of the off-shell nucleon structure functions in Eqs.(21) and nucleon momentum distribution functions $D(y, p^2)$:

$$g_1^A(x) = \int dp^2 \int_x \frac{dy}{y} \, D_1(y, p^2) \, g_1^N\left(\frac{x}{y}, p^2\right), \tag{25a}$$

$$g_T^A(x) = \int dp^2 \int_x \frac{dy}{y} \left[D_T(y, p^2) \, g_T^N\left(\frac{x}{y}, p^2\right) \; + \; D_{T2}(y, p^2) \, g_2^N\left(\frac{x}{y}, p^2\right) \right], \tag{25b}$$

where $y = (p_0 + p_z)/M$ is the fraction of the light-cone momentum of the nucleus carried by the interacting nucleon. The nucleon distribution functions are given by:

$$D_1(y, p^2) = \int \frac{d\varepsilon d^3\mathbf{p}}{(2\pi)^4} \; \mathrm{tr}\left[\mathcal{P}^{\parallel}(\varepsilon, \mathbf{p}) \left(\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z\right)\right] \delta\left(y - \frac{M + \varepsilon + p_z}{M}\right) \delta\left(p^2 - \mu^2\right), \tag{26a}$$

$$D_T(y, p^2) = \int \frac{d\varepsilon d^3\mathbf{p}}{(2\pi)^4} \; \mathrm{tr}\left[\mathcal{P}^{\perp}(\varepsilon, \mathbf{p}) \; \widehat{\mathcal{S}}_\perp\right] \delta\left(y - \frac{M + \varepsilon + p_z}{M}\right) \delta\left(p^2 - \mu^2\right), \tag{26b}$$

$$D_{T2}(y, p^2) = \int \frac{d\varepsilon d^3\mathbf{p}}{(2\pi)^4} \; \mathrm{tr}\left[\mathcal{P}^{\perp}(\varepsilon, \mathbf{p}) \; \widehat{\mathcal{T}}_2\right] \delta\left(y - \frac{M + \varepsilon + p_z}{M}\right) \delta\left(p^2 - \mu^2\right), \tag{26c}$$

where $\mu^2 \equiv M^2 + 2M(\varepsilon - \mathbf{p}^2/2M)$. We observe that g_1^A is expressed entirely in terms of g_1^N, while g_T^A receives contributions from g_T^N as well as from g_2^N. Consequently, the g_2^A structure function will receive contributions from g_1^N in addition to g_2^N.

IV. EXPANSION FORMULAE

The behavior of the nucleon distribution functions in Eqs.(26) is governed by the nuclear spectral function $\mathcal{P}(\varepsilon, \mathbf{p})$. The region of importance for $\mathcal{P}(\varepsilon, \mathbf{p})$ is $|\mathbf{p}| \lesssim p_F, \; |\varepsilon| \lesssim p_F^2/2M$, where p_F is a characteristic momentum which determines the momentum distribution of nucleons in the nucleus. For heavy nuclei this is the Fermi–momentum, $p_F \approx 300\,\mathrm{MeV}$. For a light nucleus, such as the deuteron, the analogous parameter can be determined from the average kinetic energy T as $\sqrt{M\,T} \approx 140\,\mathrm{MeV}$. Therefore the nucleon distribution functions

(26) are strongly peaked about the light-cone momentum fraction $y = 1$ and the on-mass-shell point $p^2 = M^2$.

This property of the distribution functions allows us to obtain approximate expressions for the nuclear structure functions in Eqs.(25). The p^2 dependence of the off-shell structure functions can be first approximated by expanding $g_{1,T}^N(x/y, p^2)$ in a Taylor series around $p^2 = M^2$:

$$g_{1,T}^N\left(\frac{x}{y}, p^2\right) \approx g_{1,T}^N\left(\frac{x}{y}\right) \; + \; (p^2 - M^2)\, \frac{\partial g_{1,T}^N(x/y, p^2)}{\partial p^2}\bigg|_{p^2=M^2}, \tag{27}$$

where $g_{1,T}^N(x/y) \equiv g_{1,T}^N(x/y, p^2 = M^2)$ is the structure function of the (physical) on-mass-shell nucleon (see Eqs.(8) and (21)). Expanding $g_{1,T}^N(x/y)/y$ in Eqs.(25) around $y = 1$, integrating the result term by term, and keeping terms up to order ε/M and $\mathbf{p}^2/M^2$, we obtain simple expansion formulae similar to those used in the analysis of unpolarized nuclear structure functions [30,21,25]. For the structure function g_1^A we find:

$$\frac{1}{A} g_1^A(x) \approx C_1^{(0)}\, g_1^N(x) + C_1^{(1)} \left(x g_1^N(x)\right)' + C_1^{(2)} x \left(x g_1^N(x)\right)'' + C_1^{(3)}\, \frac{\partial g_1^N(x, p^2)}{\partial \ln p^2}\bigg|_{p^2=M^2}, \tag{28}$$

where the derivatives are taken with respect to x, and the coefficients are:

$$C_1^{(0)} = \left\langle \widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z \right\rangle^{\parallel}, \tag{29a}$$

$$C_1^{(1)} = \left\langle \left(\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z\right) \left(\frac{p_z^2}{M^2} - \frac{p_z + \varepsilon}{M}\right) \right\rangle^{\parallel}, \tag{29b}$$

$$C_1^{(2)} = \left\langle \left(\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z\right) \frac{p_z^2}{2M^2} \right\rangle^{\parallel}, \tag{29c}$$

$$C_1^{(3)} = \left\langle \left(\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z\right) \frac{2}{M} \left(\varepsilon - \frac{\mathbf{p}^2}{2M}\right) \right\rangle^{\parallel}, \tag{29d}$$

with $\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z$ obtained from Eqs.(18). Here the averaging $\langle \cdots \rangle$ denotes:

$$\langle \mathcal{O} \rangle^{\parallel} \equiv \frac{1}{A} \int \frac{d\varepsilon d^3\mathbf{p}}{(2\pi)^4}\, \mathrm{tr}\left[\mathcal{P}^{\parallel}(\varepsilon, \mathbf{p})\, \mathcal{O}\right]. \tag{30}$$

Analogously, the structure function g_T^A is given by:

$$\begin{aligned}\frac{1}{A} g_T^A(x) \approx\; & C_T^{(0)}\, g_T^N(x) \; + \; C_T^{(1)} \left(x g_T^N(x)\right)' \; + \; C_T^{(2)}\, x \left(x g_T^N(x)\right)'' \; + \; C_T^{(3)}\, \frac{\partial g_T^N(x, p^2)}{\partial \ln p^2}\bigg|_{p^2=M^2} \\ & + C_{T2}^{(0)}\, g_2^N(x) \; + \; C_{T2}^{(1)} \left(x g_2^N(x)\right)'. \end{aligned} \tag{31}$$

The coefficients $C_T^{(i)}$ are identical to $C_1^{(i)}$ except for the replacements $\langle \mathcal{O} \rangle^{\parallel} \to \langle \mathcal{O} \rangle^{\perp}$ and $\widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z \to \widehat{\mathcal{S}}_{\perp}$, and the coefficients $C_{T2}^{(i)}$ are:

$$C_{T2}^{(0)} = \left\langle \widehat{\mathcal{T}}_2 \right\rangle^{\perp}, \tag{32a}$$

$$C_{T2}^{(1)} = -\left\langle \widehat{\mathcal{T}}_2\, \frac{p_z}{M} \right\rangle^{\perp}. \tag{32b}$$

We stress that the matrix elements $C_1^{(i)}$ are calculated for longitudinally polarized targets, while for $C_T^{(i)}$ and $C_{T2}^{(i)}$ the target is polarized in a direction transverse to the photon direction. Once g_1^A and g_T^A are calculated, the difference can be taken and the approximate result for g_2^A obtained. Corrections to $g_{1,2}^A$ evaluated via Eqs.(28) and (31) are of order $\mathbf{p}^3/M^3$ or higher.

In the derivation of Eqs.(28) and (31) one neglects the lower limit in the y-integration in Eqs.(25), namely the condition $x/y \leq 1$. From Eqs.(26) one can easily see that this condition gives practically no restriction on the integration region in (29,32) for $1 - x > p_F/M$. For heavy nuclei, the expansion formulae can be used safely up to $x \approx 0.7$. For light nuclei such as the deuteron, which we consider in the next section, Eqs.(28) and (31) are expected to be reliable for $x \lesssim 0.8$.

V. THE DEUTERON

In this section we consider the application of the results of Sections III and IV to the case of a deuterium nucleus. As mentioned in Section I, deep-inelastic scattering from polarized deuterons is an important means of obtaining information on the polarized neutron structure functions $g_{1,2}^n$. Extracting information on $g_{1,2}^n$ from deuterium data is only meaningful, however, if one has a reliable method of subtracting the relevant nuclear effects.

The several previous attempts to account for nuclear effects in the deuteron (for the g_1^D structure function) have not always yielded consistent results. Some early attempts [12] were made within a time-ordered framework in the infinite momentum frame, which unfortunately in practice was problematic due to the lack of knowledge about deuteron wavefunctions in this frame. Subsequent analyses [13] utilized convolution formulae obtained in direct analogy with the convolution model for unpolarized scattering. Namely, a one-dimensional convolution formula was used with the same non-relativistic "flux factor", $(1 + p_z/M)$, as appears in the unpolarized deuteron F_{2D} structure function [21,25,31,32]. Other attempts [14] were based on an operator product expansion at the nucleon level [22], however these led to different operators to those in Eq.(22a). It is important, therefore, to clarify which operators, or "flux factors", are relevant for g_1^D when all terms up to order $\mathbf{p}^2/M^2$ are consistently kept.

In addition to g_1^D, there is considerable practical value in understanding the nuclear effects on g_T^D (and hence g_2^D), to which little attention has been paid thus far. In view of upcoming experiments which will for the first time measure the deuteron g_2^D structure function [33], an estimate of the relevant nuclear corrections to g_T^D is urgently needed.

As a guide to evaluating the most efficient method for the nuclear data analysis, a comparison of the results for $g_{1,2}^D$ calculated using the convolution (25) and expansion (28,31) formulae will indicate the reliability of the latter approach to the deuteron, which to date has not been explicitly tested. Essentially the only ingredient needed to evaluate the coefficients in Eqs.(28) and (31), as well as the traces in the distribution functions of Eqs.(26), is the deuteron wavefunction. Before discussing the details of the deuteron case, however, we must first fix the nucleon inputs that will be used in the subsequent numerical calculations.

A. Nucleon Structure Function Input

For the structure functions and their derivatives we shall rely on experimental results where appropriate, and use model input where data are not yet available. For the proton and neutron g_1 structure functions we use the recent parametrization from Ref. [34] of the SLAC [35], EMC [1] and SMC [2] proton, the SLAC-E142 neutron (Helium-3) [8], and SMC deuteron [9] data. As an illustration of the quality of the fit, we plot in Fig.2(a) the $xg_1^{p,n}$ structure functions at $Q^2 = 4$ GeV^2, compared with the data.

For the g_2 structure function the study of nuclear effects is more problematic, since this receives contributions from both twist-2 and 3 operators, the latter of which contain quark-gluon interactions and also explicitly depend on quark masses. Following the standard decomposition of g_2^N into the different twist components, we write:

$$g_2^N(x) = g_2^{N(WW)}(x) \ + \ \overline{g}_2^N(x), \tag{33}$$

where the twist-2 part is given by the Wandura-Wilczek relation [36]:

$$g_2^{N(WW)}(x) = \int_x^1 \frac{dy}{y} \left(1 - \delta(1 - x/y)\right) \ g_1^N(y), \tag{34}$$

and satisfies the Burkhardt–Cottingham sum rule [37]:

$$\int_0^1 dx \ g_2^N(x) = 0. \tag{35}$$

The twist-3 piece ($\overline{g}_2^N$) of g_2^N is at present not very well determined at all. There is disagreement even about the magnitude and sign of its moments [38,39]. Based on a covariant parton model approach, Jackson, Roberts and Ross [40] argued in favor of a very small twist-3 component. Within bag models, on the other hand, one finds [38,41] quite a sizable $\overline{g}_2^N$ contribution compared with the Wandura-Wilczek term. An additional problem is how to relate the structure function calculated in the bag (or some other) model, which one assumes to be applicable at some low resolution scale $Q^2 = \mathcal{O}(\Lambda_{QCD}^2)$, to that appropriate to DIS experiments ($Q^2 \gtrsim 5$ GeV^2). One prescription [42] is to simply assume the validity of the Altarelli-Parisi evolution equations down to very low Q^2. Even within this pragmatic approach, while the evolution of the twist-2 component follows that of g_1, which is understood, for the $\overline{g}_2^N$ piece only the $N = 2$ and 4 moments can be handled exactly. The solution adopted in [41] was to use an approximate solution of the evolution equations that was derived in the large-N_C limit.

Since the only available data [43] on the g_2^p structure function cannot yet unambiguously discriminate between the various models of $\overline{g}_2$, we will estimate the size of the nuclear effects for several models. To cover the potential range of results for $\overline{g}_2^N$, we take $\overline{g}_2^N = 0$, as suggested by Ref. [40], and also the bag model predictions, evolved to $Q^2 \sim 5$ GeV^2 using the prescription adopted in Ref. [41]. As an indication of the possible variation of the total g_2^N structure function with different twist-3 components, we plot in Fig.2(b) g_2^N for these two cases. Also shown is the sum, $g_T^N = g_1^N + g_2^N$, which will be relevant in the actual evaluation of the nuclear structure function, Eqs.(25,28,31). One can see that while the effect of the twist-3 component is certainly not negligible, it does not alter drastically the overall shape and sign of g_2^N and g_T^N.

From the two-dimensional convolution equations (25) it is clear that a consistent description of nuclear effects to order $\mathbf{p}^2/M^2$ requires modeling in addition the p^2 dependence of the off-shell nucleon structure functions $g_{1,2}^N(x,p^2)$. To this order of accuracy this can be achieved by determining the slope with respect to p^2 at the on-mass-shell point, Eq.(27). One could, for example, formulate $g_{1,2}^N$ in terms of relativistic quark–nucleon vertex functions as described in Refs. [16,26,27,44,45], and calculate the derivative directly. Alternatively, to obtain a quick estimate of the overall order of magnitude of the off-shell effect, we can extend the model of Ref. [25], which is based on a dispersion representation of the unpolarized nucleon structure function, to the polarized case.

Within the latter approach, in the impulse approximation for the quarks the g_1^N structure function can be written:

$$g_1^N(x,p^2) = \int ds \int_{-\infty}^{k^2_{max}(x,p^2)} dk^2 \ \rho(k^2,s,p^2,x), \tag{36}$$

where $k^2_{max}(x,p^2) = -xs/(1-x) + xp^2$ is the kinematical maximum of the quark momentum squared k^2, $s = (p-k)^2$ is the center-of-mass energy squared of the "spectator" quark system, and ρ is the quark spectral function extended to the nucleon off-mass-shell region. Following Ref. [25] we assume that ρ has no explicit x-dependence, and that the spectrum in s can be approximated by that calculated for a single effective mass $\bar{s}$, $\rho \propto \delta(s-\bar{s})$. At moderate $Q^2 \sim 5-10$ GeV^2 one finds typically $\bar{s} \simeq 2$ GeV^2 [25]. Taking a factorized p^2 and k^2 dependence in ρ then gives g_1^N as a product of two functions:

$$g_1^N(x,p^2) = \varphi(p^2)\ \mathcal{F}\left(k^2_{max}(x,p^2)\right), \tag{37}$$

normalized such that $\varphi(M^2) = 1$ and hence $\mathcal{F}(k^2_{max}(x,M^2)) = g_1^N(x)$. The explicit p^2-dependence in the function $\varphi(p^2)$ is dynamical in origin, while the p^2-dependence in the function $\mathcal{F}(k^2_{max})$ that enters through the upper limit of the k^2 integration is purely kinematical. After some algebra, the derivative of $g_1^N(x,p^2)$ with respect of p^2 can then be written:

$$\left.\frac{\partial g_1^N(x,p^2)}{\partial p^2}\right|_{p^2=M^2} = g_1^N(x)\ \left.\frac{\partial\varphi(p^2)}{\partial p^2}\right|_{p^2=M^2} + \left(g_1^N(x)\right)'\ \frac{x\ (1-x)^2}{M^2\ (1-x)^2\ -\ \bar{s}}\ . \tag{38}$$

To determine the slope of the $\varphi(p^2)$ at $p^2 = M^2$ we assume that the first moment of the non-singlet part of g_1^N is not renormalized off-shell (at least in the vicinity of $p^2 \approx M^2$):

$$\int_0^1 dx\ \left.\frac{\partial g_1^N(x,p^2)}{\partial p^2}\right|_{p^2=M^2} = 0. \tag{39}$$

The motivation for this condition is that since the axial U(1) anomaly [46] is absent in the non-singlet sector, in the chiral limit the axial charge of the nucleon is a conserved quantity. For the singlet component of g_1^N little is known about how the effects of the axial anomaly extrapolate into the off-shell region, although one would not expect dramatic consequences as long as $p^2 \approx M^2$. To satisfy Eq.(39) with $\bar{s} = 2$ GeV^2 one needs a slope of approximately $\partial\varphi(p^2)/\partial p^2|_{p^2=M^2} \approx -0.16$ GeV^{-2}. For larger values of $\bar{s}$, $\bar{s} \simeq 3$ GeV^2, the slope becomes $\simeq -0.1$ GeV^{-2}.

The extension of the above off-shell model to the g_2^N structure function is more problematic. For example, there is no known justification for a normalization condition such as in Eq.(39) to be valid for g_2^N. Furthermore, there is little knowledge about how the higher twist correlation, or final state interaction, effects involving the (nucleon–quark) "spectator" system would modify the effective mass $\bar{s}$. For these reasons we postpone for the time being a more detailed discussion about the off-shell dependence in g_2^N.

With these inputs for the nucleon structure function we can now proceed to evaluate numerically the structure functions of the deuteron.

B. Convolution Results

For the deuteron, the traces in Eqs.(26) can be expressed in terms of the deuteron wavefunctions $\Psi_{m=+1}(\mathbf{p})$ with spin projection $m = +1$ along the axis of quantization (see Appendix B) [3]:

$$\text{tr}\ [\mathcal{P}(\varepsilon,\mathbf{p})\ \mathcal{O}]_D\ = \Psi^\dagger_{+1}(\mathbf{p})\left(\mathcal{O}^{(p)}+\mathcal{O}^{(n)}\right)\Psi_{+1}(\mathbf{p})\ \ 2\pi\ \delta(\varepsilon-\epsilon_D+\mathbf{p}^2/2M), \tag{40}$$

where ϵ_D is the deuteron binding energy. The expectation values in Eq.(40) with the operators $\mathcal{O} = \widehat{\mathcal{S}}_0 + \widehat{\mathcal{S}}_z$, $\widehat{\mathcal{S}}_\perp$ and $\widehat{\mathcal{T}}_2$ determine the deuteron distribution functions. From Eq.(B5) (for the axis of quantization parallel to the z-direction) we find:

$$\begin{aligned}\Psi^{(\|)\dagger}_{+1}(\mathbf{p})\left(\widehat{\mathcal{S}}_0+\widehat{\mathcal{S}}_z\right)\Psi^{(\|)}_{+1}(\mathbf{p})\ =\ & \frac{4\pi^2}{\mathbf{p}^2}\left\{\left(1-\frac{\mathbf{p}^2}{2M^2}\right)\left[u^2+\frac{uw}{\sqrt{2}}\left(1-3\widehat{p}_z^2\right)+w^2\left(\frac{3}{2}\widehat{p}_z^2-1\right)\right]\right.\\ & \left.+\ \frac{p_z}{M}\left(1+\frac{p_z}{2M}\right)\left[u-\frac{w}{\sqrt{2}}\right]^2\right\},\end{aligned} \tag{41a}$$

where $u(p)$ and $w(p)$ are the S- and D-state wavefunctions in momentum space, and $p_z = |\mathbf{p}|\widehat{p}_z$, with $\widehat{p}_z\ =\ \cos\theta$ (θ is the angle between $\widehat{\mathbf{p}}$ and the z-axis).

For a deuteron polarized in a direction transverse relative to the photon direction, $S\ =\ S^\perp$, we have:

$$\begin{aligned}\Psi^{(\perp)\dagger}_{+1}(\mathbf{p})\,\widehat{\mathcal{S}}_\perp\,\Psi^{(\perp)}_{+1}(\mathbf{p})\ =\ & \frac{4\pi^2}{\mathbf{p}^2}\left\{\left(1-\frac{\mathbf{p}^2}{2M^2}\right)\left[u^2+\frac{uw}{\sqrt{2}}\left(1-3(\widehat{\mathbf{p}}_\perp\cdot\mathbf{S}^\perp)^2\right)\right.\right.\\ & \left.\left.+w^2\left(\frac{3}{2}(\widehat{\mathbf{p}}_\perp\cdot\mathbf{S}^\perp)^2-1\right)\right]+\frac{\left(\mathbf{p}_\perp\cdot\mathbf{S}^\perp\right)^2}{2M^2}\left[u-\frac{w}{\sqrt{2}}\right]^2\right\},\end{aligned} \tag{41b}$$

$$\begin{aligned}\Psi^{(\perp)\dagger}_{+1}(\mathbf{p})\,\widehat{\mathcal{T}}_2\,\Psi^{(\perp)}_{+1}(\mathbf{p})\ =\ & -\frac{4\pi^2}{|\mathbf{p}|}\frac{\left(\widehat{\mathbf{p}}_\perp\cdot\mathbf{S}^\perp\right)^2}{M}\left\{\frac{3}{2}\left(1-\frac{p_z}{M}\right)\widehat{p}_z w\left(w-\sqrt{2}u\right)\right.\\ & \left.+\ \frac{|\mathbf{p}|}{M}\left(u-\frac{w}{\sqrt{2}}\right)^2\right\}.\end{aligned} \tag{41c}$$

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[3]Note that the argument in the deuteron wavefunction is the relative nucleon momentum, which in the deuteron rest frame coincides with the single particle momentum $\mathbf{p}$.

Substituting these results into Eqs.(26), we can now evaluate the $g_{1,T}^D$ structure functions of the deuteron. In Fig.3 we plot the ratio g_1^D/g_1^N as a function of x, for $Q^2 = 4$ GeV^2, using wavefunctions from several different models of deuteron structure [47–49]. The differences between the curves at small x are due mainly to the different deuteron D-state probabilities, namely 5.8% for the Paris, 4.3% for the Bonn (full model) and 4.7% for the Buck/Gross (pseudo-vector coupling) models. In calculating the curves in Fig.3 we have made the approximation $g_1^N(x,p^2) \approx g_1^N(x,M^2)$. Within the model of Ref. [16] it was found that this approximation is good to within 0.5% for $x \lesssim 0.8$. We shall illustrate the effects of the $\partial g_1^N/\partial p^2$ term evaluated using the off-shell model of Section V A in the next section.

The transverse structure function ratio, g_T^D/g_T^N, is shown in Fig.4, at $Q^2 = 4$ GeV^2, calculated for the Paris wavefunction [47] (solid curve) [4]. The result turns out to be quite similar to the ratio of the g_1 structure functions in Fig.3 (dashed curve), the main difference being at large x, where the g_T ratio rises above unity earlier. The faster rise is even more pronounced for the ratio of the twist-2 components of g_T (dotted curve). At intermediate x values ($x \lesssim 0.5$) the g_T^D/g_T^N ratio appears mostly independent of the model for g_2^N, so that one may reasonably safely extract information on g_T^N (and hence g_2^N) from the transverse deuteron data.

C. Expansion Results

In the range of x at which most of the data are taken, namely $x \lesssim 0.6$, the arguments given in Section IV would suggest that the expansion formulae in Eqs.(28) and (31) should be excellent approximations to the full convolution results. Since these would simplify the analysis of the deuterium data, one may then take advantage of the simple expansion approximations in this region of x.

To test the reliability of the expansion approach, we must calculate the coefficients $C_1^{(i)}$, $C_T^{(i)}$ and $C_{T2}^{(i)}$ for a deuteron target. Using the matrix element in Eq.(41a) we obtain explicit expressions for the coefficients in Eqs.(29) of the various terms in the structure function $g_1^D(x)$:

$$C_1^{(0)} = 1 - \frac{3}{2}P_D \;-\; \frac{2}{3M}\left(T_0 + \frac{1}{\sqrt{2}}T_{02} - T_2\right), \tag{42a}$$

$$C_1^{(1)} = \left(1 - \frac{3}{2}P_D\right)\frac{|\epsilon_D|}{M} \;+\; \frac{1}{M}\left(T_0 + \frac{2\sqrt{2}}{5}T_{02} - \frac{9}{10}T_2\right), \tag{42b}$$

$$C_1^{(2)} = \frac{1}{3M}\left(T_0 - \frac{2\sqrt{2}}{5}T_{02} - \frac{1}{10}T_2\right), \tag{42c}$$

$$C_1^{(3)} = -\frac{2}{M}\left[\left(1 - \frac{3}{2}P_D\right)|\epsilon_D| + 2T_0 - T_2\right], \tag{42d}$$

[4]Note that plotting the ratio of g_2 structure functions is not very instructive since g_2^N changes sign — to obtain the nuclear effects on g_2 alone one can subtract the g_1 component determined above.

where $P_D = \int dp\, w^2(p)$ is the D-state probability in the deuteron, with $p \equiv |\mathbf{p}|$ (not to be confused with the momentum four-vector p used above). Here T_0, T_2 and T_{02} represent the average nucleon kinetic energies associated with each component of the deuteron wavefunction:

$$T_0 = \int_0^\infty dp\, \frac{\mathbf{p}^2}{2M}\ u^2(p), \tag{43a}$$

$$T_2 = \int_0^\infty dp\, \frac{\mathbf{p}^2}{2M}\ w^2(p), \tag{43b}$$

$$T_{02} = \int_0^\infty dp\, \frac{\mathbf{p}^2}{2M}\ u(p)\ w(p). \tag{43c}$$

To illustrate the role of the various terms in the expansion we plot in Fig.5 the zeroth order contribution, proportional to $C_1^{(0)}$, together with the higher order terms, scaled by a factor 100. Evident at large values of x ($x \sim 0.8$) is the role of the second derivative term, proportional to $C_1^{(2)}$, which gives the characteristic rise (due to Fermi motion) of the structure function ratio g_1^D/g_1^N as $x \to 1$, see Fig.6. The trough in the ratio at $x \sim 0.6$ arises from the single differential term proportional to $C_1^{(1)}$ which is large and negative in this region. Although it does not give rise to any uniquely distinct features in the structure function ratio, it is clear that the off-shell component (dotted curve in Fig.5) is of the same order of magnitude as the other higher order corrections, and must be included in any precision analysis of nuclear effects in the deuteron.

In Fig.6 we compare the performance of the expansion formula for the ratio g_1^D/g_1^N with several other methods of computation, neglecting for the moment the off-shell contributions. The simplest approach (and the one used by the SMC in their analysis [9]) is to use a constant depolarization factor, $(1 - 3/2\, P_D)$, which roughly corresponds to the first term in the Taylor series in Eq.(28). In fact, the order $\mathbf{p}^2/M^2$ correction to $(1 - 3/2\, P_D)$ is of the order of 10%, i.e. an overall correction to the structure function ratio of $\sim 0.5\%$. At present this is still smaller than the uncertainty in P_D between the different models. Compared with the convolution results, one sees that the expansion formula works remarkably well for $x \lesssim 0.7$, where the two results are almost indistinguishable. As expected, for $x \gtrsim 0.7$ the expansion curve overshoots the convolution result, which is understood from the fact that in the expansion formula one is neglecting the lower limit of integration, namely x, of $D(y)$ over y, and replacing it by zero.

To examine the effect of the off-shell $C_1^{(3)}$ term in Eq.(28) on the g_1 ratio, we plot in Fig.7 the ratio with $\partial g_1^N/\partial p^2 = 0$ (dashed) and with the slope determined through Eq.(38) (solid) with the mass parameter $\bar{s} = 2$ GeV^2. The overall effect on the shape of the ratio is quite minimal, and the trend follows the shape of the off-shell component in Fig.5. Note that within this model the off-shell curve at large x ($x \gtrsim 0.7$) approaches the on-shell limit, although in this region the expansion approximation itself is no longer accurate. Within the relativistic model of Ref. [16] (dotted curve), the magnitude of the (negative) off-shell effects was found to increase rapidly beyond $x \sim 0.8$, which is a further reason why the expansion curves tend to be larger at very large x. In the intermediate-x region, on the other hand, there is little to distinguish all of the curves for $0.2 \lesssim x \lesssim 0.7$.

For the transversely polarized function $g_T^D(x)$, using Eqs.(41b,41c) we obtain:

$$C_T^{(0)} = C_1^{(0)}, \tag{44a}$$

$$C_T^{(1)} = \left(1 - \frac{3}{2}P_D\right)\frac{|\epsilon_D|}{M} + \frac{5}{3M}\left(T_0 + \frac{2\sqrt{2}}{25}T_{02} - \frac{29}{50}T_2\right), \tag{44b}$$

$$C_T^{(2)} = \frac{1}{3M}\left(T_0 + \frac{\sqrt{2}}{5}T_{02} - \frac{7}{10}T_2\right), \tag{44c}$$

$$C_T^{(3)} = C_1^{(3)}, \tag{44d}$$

$$C_{T2}^{(0)} = -\frac{2}{3M}\left(T_0 - \frac{7\sqrt{2}}{10}T_{02} + \frac{1}{5}T_2\right), \tag{44e}$$

$$C_{T2}^{(1)} = \frac{1}{5M}\left(T_2 - \sqrt{2}T_{02}\right). \tag{44f}$$

The various components of xg_T^D in Eq.(31) are shown in Fig.8. As in Fig.6, the higher order corrections are scaled by 100. (As discussed in Section V A, we do not consider the off-shell $C_T^{(3)}$ term here.) While small, the corrections proportional to xg_2^N and its derivative are still of the same order of magnitude as the g_T^N-dependent terms.

Finally, the relevant quantity needed for the extraction of the transverse nucleon structure function from g_T^D is plotted in Fig.9 (solid curves). The ratio g_T^D/g_T^N comes out to be very similar to the one obtained from the full convolution model (dashed curves). In Fig.9 curves (i) have $\overline{g}_2^N = 0$, while curves (ii) include the twist-3 component. As a reference point, the result with a constant depolarization factor, $(1 - 3/2\, P_D)$, is also shown (dotted line). These results indicate that the expansion formula (31) is quite a good approximation to the convolution model (25b) over nearly the entire x-domain of current experiments.

VI. CONCLUSION

We have presented a formulation of spin-dependent deep-inelastic scattering from spin 1/2 and 1 nuclear targets. Starting from a covariant framework we have derived non-relativistic convolution formulae for the nuclear g_1^A and $g_T^A(\equiv g_1^A + g_2^A)$ structure functions in terms of polarized nucleon distribution functions and off-mass-shell extrapolations of the nucleon structure functions $g_{1,T}^N$. It is known that relativistically the factorization of nuclear and nucleon parts of the total structure function, which is necessary for convolution, does not hold. Our results, however, are self-consistent to order $\mathbf{p}^2/M^2$ in the nucleon momentum, and represent the first systematic derivation of convolution formulae for polarized structure functions of weakly bound nuclei. To this order, while g_1^A can be expressed in terms of g_1^N, we find that g_2^A receives contributions from both the g_2^N *and* g_1^N structure functions of the nucleon.

We have further utilized the fact that the nucleon momentum distributions in non-relativistic nuclei are strongly peaked around the light-cone momentum fraction $y \sim 1$ and the on-mass-shell point $p^2 \sim M^2$. Expanding the virtual nucleon structure functions about these points we obtained simple expansion formulae for $g_{1,T}^A$ valid for $x \lesssim 0.7$.

The performance of the convolution and expansion approaches was examined for the case of the deuteron, which has direct practical implications for the extraction of the free neutron structure function from deuterium data. For the g_1^D structure function good agreement was found between the expansion approximation and the non-relativistic convolution for all x below ~ 0.6. Differences between the non-relativistic convolution and previous relativistic

calculations become noticeable only for $x \gtrsim 0.7$. At smaller x the source of the largest uncertainty is the non-relativistic deuteron D-state probability.

We have also investigated for the first time the nuclear effects relevant for the extraction of the neutron structure function g_T^n (or g_2^n) from measurements of the transverse structure function of the deuteron g_T^D. Qualitatively these were found to be similar to the nuclear effects for g_1^D, and for $x \lesssim 0.5$ largely independent of the details of the twist-3 component of g_2^N.

We can conclude, therefore, that for both g_1^D and g_T^D the non-relativistic expansion formula provides a simple and, within current experimental accuracy, reliable means of analyzing nuclear effects in the deuteron. In future high-precision experiments [50,51] the role of relativistic corrections as well as corrections to the impulse approximation itself may be more significant. These experiments should provide us with valuable guidance as to the relevance of relativistic effects in light nuclei, and where the impulse approximation may break down.

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APPENDIX A: NON-RELATIVISTIC REDUCTION OF THE NUCLEON FIELD OPERATOR

To derive the relation in Eq.(12) between the relativistic and non-relativistic field operators, we first write the relativistic nucleon field operator N in terms of upper and lower components, φ and χ, respectively:

$$N(\mathbf{r}, t) = e^{-iMt} \begin{pmatrix} \varphi(\mathbf{r}, t) \\ \chi(\mathbf{r}, t) \end{pmatrix}. \tag{A1}$$

In the extreme non-relativistic limit $\mathbf{p} \to 0$, $\chi \to 0$, and at zeroth order in $|\mathbf{p}|/M$, the non-relativistic nucleon field is simply given by the "large" upper component φ. We require, however, a relation which is valid to order $\mathbf{p}^2/M^2$.

The equation of motion for a nucleon field in the presence of a "potential" $\widehat{\mathcal{V}}$ can be written:

$$(i\not{\partial} - M)\, N = \widehat{\mathcal{V}}, \tag{A2}$$

where

$$\widehat{\mathcal{V}} = S + \not{V} + i\gamma_5 P + \gamma_5 \not{A} \tag{A3}$$

describes the interaction of a single nucleon with mesonic fields produced by the surrounding nucleons. The first two terms in Eq.(A3) describe the coupling of a nucleon to scalar and

vector meson fields, $S = g_s\sigma$ and $V_\mu = g_\omega\omega_\mu$, respectively, while the last two correspond to pseudoscalar (PS) and pseudovector (PV) πN-couplings, $P = g_\pi\pi$ and $A_\mu = (g'_\pi/2M)\ \partial_\mu\pi$ (for simplicity we ignore isospin).

Written for the upper and lower components, the Dirac equation (A2) reads:

$$(i\partial_0 - S - V_0 - \boldsymbol{\sigma}\cdot\boldsymbol{A})\,\varphi \ - \ (\boldsymbol{\Pi}\cdot\boldsymbol{\sigma} + iP - A_0)\,\chi = 0, \tag{A4a}$$

$$(\boldsymbol{\Pi}\cdot\boldsymbol{\sigma} - iP - A_0)\,\varphi \ - \ (i\partial_0 + 2M - S + V_0 + \boldsymbol{\sigma}\cdot\boldsymbol{A})\,\chi = 0, \tag{A4b}$$

where $\boldsymbol{\Pi} \equiv \mathbf{p} - \boldsymbol{V}$, and $\mathbf{p} = -i\boldsymbol{\nabla}$ is the momentum operator. In the non-relativistic limit the driving term in Eq.(A4b) is $2M$. Expanding χ as a series in inverse powers of M gives:

$$\chi = \left(1 - \frac{i\partial_0 + S - V_0 - \boldsymbol{\sigma}\cdot\boldsymbol{A}}{2M} + \ldots\right)\frac{(\boldsymbol{\Pi}\cdot\boldsymbol{\sigma} - iP - A_0)\,\varphi}{2M}, \tag{A5}$$

where the leading term is of order $|\mathbf{p}|/M$, while the next term is of order $|\mathbf{p}|^3/M^3$. For weakly bound nuclei, such as ^{2}H or ^{3}He, the interaction $\widehat{\mathcal{V}}$ is of the same order as the kinetic energy, $\mathbf{p}^2/2M$. Furthermore, the kinetic and potential energies are almost equal in magnitude while opposite in sign [5], so that one can treat the different parts of the interaction (S, V_0 and $\boldsymbol{\sigma}\cdot\boldsymbol{A}$) as all being of the order $\mathbf{p}^2/M^2$. To this order, it is sufficient therefore to keep only the first term in the first parentheses in Eq.(A5).

From the equations of motion for the mesonic field we observe that the ratio of the spatial ($\boldsymbol{V}$) to time (V_0) components of the vector current is $\left|\bar{N}\boldsymbol{\gamma}N\right|\big/\,\bar{N}\gamma_0 N \sim |\mathbf{p}|\,/\,M$, so that $|\boldsymbol{V}/V_0| \sim |\mathbf{p}|/M$. Furthermore, since the time component of the axial vector interaction is given by the time derivative of the pion field, $A_0 = (g'_\pi/2M)\ \partial_0\pi$, it is reasonable to assume that $A_0 \sim (\mathbf{p}^2/M^2)P$. Therefore to order $\mathbf{p}^2/M^2$ one can neglect $\boldsymbol{V}$ and A_0 in Eq.(A5), so that the lower component χ can be written:

$$\chi = \frac{1}{2M}\,(\boldsymbol{\sigma}\cdot\mathbf{p} - iP)\,\varphi. \tag{A6}$$

Substituting Eq.(A6) into Eq.(A4a) leads then to the Pauli–Schrödinger equation for the two-component nucleon spinor:

$$i\partial_0\varphi = \left(\frac{\mathbf{p}^2}{2M} + V_{\rm NR}\right)\varphi, \tag{A7}$$

$$V_{\rm NR} = S \ + \ V_0 \ + \ \boldsymbol{\sigma}\cdot\boldsymbol{A} \ + \ \frac{1}{2M}\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}P) \ + \ \frac{P^2}{2M}\ , \tag{A8}$$

where $V_{\rm NR}$ is the non-relativistic analog of $\widehat{\mathcal{V}}$. Recalling that we assume all parts of the interaction being of the same order as the kinetic energy, $\mathbf{p}^2/2M$, we observe from Eq.(A8) that the PS term should be of order $P \sim \boldsymbol{\sigma}\cdot\mathbf{p}$, and so must be kept in Eq.(A6).

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[5]Note however that in relativistic models of nuclear matter, such as the Walecka model, some parts of the interaction $\widehat{\mathcal{V}}$ can be large but of opposite sign (e.g. S and V_0), leading to a small overall $\widehat{\mathcal{V}}$.

Note also that to order $\mathbf{p}^2/M^2$ the nucleon density $N^\dagger N$ receives contributions from the lower component, χ, in which case φ cannot be identified with the properly normalized non-relativistic nucleon field ψ. Following Ref. [28], we introduce a renormalization constant Z such that $\varphi = Z\psi$, with Z determined by the particle number (charge) conservation condition:

$$\int d^3\mathbf{r}\,\psi^\dagger\psi = \int d^3\mathbf{r}\,N^\dagger N\ . \tag{A9}$$

Inserting χ in Eq.(A6) into (A9) we find, to order $\mathbf{p}^2/M^2$:

$$Z = 1 - \frac{1}{8M^2}\left(\mathbf{p}^2\ +\ \boldsymbol{\sigma}\boldsymbol{\cdot}(\boldsymbol{\nabla}P)\ +\ P^2\right), \tag{A10}$$

$$N(\mathbf{r},t) = e^{-iMt}\begin{pmatrix} Z\,\psi(\mathbf{r},t) \\ \dfrac{(\boldsymbol{\sigma}\boldsymbol{\cdot}\mathbf{p}-iP)}{2M}\,\psi(\mathbf{r},t)\end{pmatrix}. \tag{A11}$$

Therefore the renormalization constant depends explicitly on the PS pion–nucleon interaction. In the non-relativistic limit the PV and PS couplings result in identical P-wave pion-nucleon interactions, as seen from Eq.(A8). However, the PS coupling also generates a strong $S-$wave πN interaction (the term $P^2/2M$ in Eq.(A8)). Considered alone, the PS term leads to incorrect πN scattering lengths, and its contribution is only cancelled by the introduction of a non-linear $\sigma\pi\pi$ coupling. In our model therefore we consider only the PV coupling, which does not lead to the spurious $S-$wave interaction. In this case the non-relativistic expression for the four-component nucleon spinor is given by Eq.(12), and the renormalization constant is interaction-independent.

APPENDIX B: DEUTERON IDENTITIES

For completeness we present here some definitions and useful identities for the deuteron which are used in Section V A.

The deuteron wavefunction in momentum space is defined as:

$$\Psi_m(\mathbf{p}) = \frac{\sqrt{2\pi^2}}{p}\left(u(p) - w(p)\,\frac{S_{12}(\widehat{\mathbf{p}})}{\sqrt{8}}\right)\chi_{1,m}, \tag{B1}$$

where $p = |\mathbf{p}|$ and m is the projection of the deuteron spin on the axis of quantization, $\chi_{1,m}$ is the spin 1 wavefunction of the two-nucleon system, and $S_{12}(\widehat{\mathbf{p}})$ is the tensor operator with $\widehat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. We define the S- and D-state wavefunctions in momentum space as:

$$\sqrt{2\pi^2}\,u(p) = \int_0^\infty dr\,pr\,j_0(pr)\,u(r), \tag{B2a}$$

$$\sqrt{2\pi^2}\,w(p) = \int_0^\infty dr\,pr\,j_2(pr)\,w(r), \tag{B2b}$$

where $u(r)$ and $w(r)$ are the standard wavefunctions in configuration space, and normalized such that:

$$\int_0^\infty dp\ \left(u^2(p) + w^2(p)\right) = 1. \tag{B3}$$

If $\mathbf{S}$ is a unit vector along the direction of the spin quantization, then

$$\frac{1}{2}\chi_{1,m}^{\dagger}\left(\boldsymbol{\sigma}^{(p)}+\boldsymbol{\sigma}^{(n)}\right)\chi_{1,m}=m\mathbf{S}, \tag{B4}$$

where $\boldsymbol{\sigma}^{(p),(n)}$ are SU(2) Pauli spin matrices acting on the proton and neutron wave function respectively. Using some simple relations from the SU(2) algebra we find:

$$\Psi_m^{\dagger}(\mathbf{p})\left(\boldsymbol{\sigma}^{(p)}+\boldsymbol{\sigma}^{(n)}\right)\Psi_m(\mathbf{p})=m\,\frac{4\pi^2}{\mathbf{p}^2}\left[\mathbf{S}\left(u^2+\frac{uw}{\sqrt{2}}-w^2\right)+\frac{3}{2}\widehat{\mathbf{p}}\left(\widehat{\mathbf{p}}\cdot\mathbf{S}\right)w\left(w-\sqrt{2}u\right)\right]. \tag{B5}$$

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FIGURES

FIG. 1. Deep inelastic scattering from a polarized nucleus in the impulse approximation. The momenta of the target nucleus (P), virtual nucleon (p) and photon (q) are marked, and S denotes the nuclear spin vector.

FIG. 2. Nucleon structure function input used in the calculation of nuclear structure functions, for $Q^2 = 4$ GeV^2: (a) parametrization [34] of the proton [1,2,35] and neutron [3] xg_1 data; (b) isoscalar nucleon structure functions $xg_1^N\,(=(xg_1^p + xg_1^n)/2)$ (solid), xg_2^N (dashed), and $xg_T^N\left(= xg_1^N + xg_2^N\right)$ (dotted). In (b) curves (1) contain the twist-2 component of g_2^N only, while in curves (2) g_2^N has in addition a twist-3 contribution based on the bag model calculation of Ref. [41].

FIG. 3. Ratio of the deuteron to nucleon g_1 structure functions calculated via the convolution formula in Eq.(25a), with the Paris [47] (solid), Bonn [48] (dotted) and Gross [49] (dashed) deuteron wavefunctions. The latter, which also include small P-state components, are renormalized so that the S- and D-state wavefunctions alone are normalized to unity.

FIG. 4. Ratio of the transverse g_T deuteron to nucleon structure functions within the convolution model, Eq.(25b), for the Paris wavefunction [47], with (solid) and without (dotted) the twist-3 component of g_2^N. For comparison the g_1^D/g_1^N ratio (dashed) from Fig.3 is also shown.

FIG. 5. Contributions to the deuteron xg_1^D structure function from various terms in the expansion formula, Eq.(28): zeroth order term (solid), first derivative (dashed), second derivative (dot-dashed), nucleon off-shell contribution (dotted). The latter three higher order terms are scaled by a factor 100.

FIG. 6. Deuteron to nucleon structure function ratio, g_1^D/g_1^N, using the expansion formula (solid), compared with the convolution result (dashed) from Fig.3, and with a constant depolarization factor $(1 - 3/2\,P_D)$ (dotted), with $P_D \simeq 5.8\%$ from the Paris wavefunction [47].

FIG. 7. Deuteron to nucleon g_1 structure function ratio from the non-relativistic expansion formula with (solid) and without (dashed) the off-shell component in Eq.(28), for the deuteron wavefunction of Ref. [49]. Shown also is the result of the relativistic calculation of Ref. [16] (dotted).

FIG. 8. Contributions to the xg_T^D structure function in the expansion formula, Eq.(31): (a) xg_T^N components — zeroth order term (solid), first (dashed) and second (dot-dashed) derivatives; (b) xg_2^N (triple-dot–dashed) and $(xg_2^N)'$ (dashed) components, on the background of the zeroth order term (solid) in (a).

FIG. 9. Transverse deuteron to nucleon structure function ratio using the expansion formula (solid), Eq.(31), compared with the convolution model result (dashed). Curves (i) have $\overline{g}_2^N = 0$, while curves (ii) include the twist-3 component. The dotted curve indicates the constant depolarization ratio, $(1 - 3/2\, P_D)$, with $P_D \simeq 5.8\%$ from the Paris potential [47].